NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSAM	**LEVEL: 7**
COURSE CODE: CAN702S	**COURSE NAME:** COMPLEX ANALYSIS
SESSION: JANUARY 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS: 100**

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	DR. NEGA CHERE
MODERATOR:	PROF. FORTUNE' MASSAMBA

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided. 2. Show clearly all the steps used in the calculations. 3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1 [17]

1.1. Determine the imaginary part of $f(z) = \frac{1}{1-\bar{z}}$ where z = x + iy. [7]

1.2. Use exponential form to express $(-1 + i)^{18}$ in the form of x + iy. [10]

QUESTION 2 [10]

Show that $f(z) = \bar{z}$ is nowhere differentiable.

QUESTION 3 [7]

Find the image of the set $\{re^{i\theta} : 2 < r < 4$ and $\frac{\pi}{2} < \theta < \frac{3\pi}{2}\}$. under the mapping $w = z^{\frac{1}{2}}$. Sketch properly both regions.

QUESTION 4 [30]

4.1. For which values of z does the function $f(z) = (z - \bar{z})(z - 1)$ satisfy the Cauchy-Riemann equations? [13]

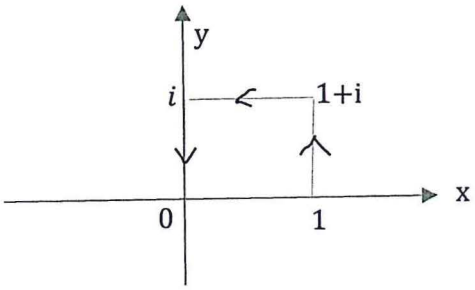
4.2. Show that the function $u(x, y) = xy^3 - x^3y + 2x$ is harmonic and determine the harmonic conjugate $v(x, y)$, with $v(0,0) = 0$. [17]

QUESTION 5 [24]

Compute the following integrals and write the most simplified answer.

5.1. $\int_i^{1+i} \left(z^2 + \frac{1}{z}\right) dz$. [7]

5.2. Evaluate $\int_C y\, dz$ where C is the polygonal path with vertices $1, 1 + i, i, 0$ as shown in the figure below. [17]



QUESTION 6 [12]

6.1. $\int_C \frac{z^3}{z^2+2z-3} dz$ where C is the circle $|z| = 2$ traversed once counterclockwise direction. [7]

6.2. $\int_C \frac{e^{z^2}}{z^2+9} dz$ where C is the circle $|z| = 2$ oriented counterclockwise. [5]

END OF SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION PAPER